Chapter 3
Discount Rates For Intangible Asset Related Profit Flows

I. The Cost of Capital, Discount Rate, and Required Rate of Return

The terms "cost of capital," "discount rate," and "required rate of return" all mean the same thing. The basic idea is simple – a capital investment of any kind, including intangible capital, represents foregone consumption today in return for the likelihood of more consumption tomorrow. The required rate of return to a capital investment is just that – the rate of return, r, on \$1 of today's foregone consumption (*i.e.*, investment) at which one would be indifferent between consuming \$1 today and consuming \$1 times $(1+r)$ tomorrow.

We have already introduced the reader to two types of required rate of return: 1) the required rate of return to routine invested capital, and 2) the required rate of return to intangible asset investments, $\bar{r}$ (also known as the cost of capital or discount rate for a residual profit flow). In this chapter, we introduce two other important required rates of return, related to two other types of intangible asset-related profit flows: 1) the required rate of return for a licensee's operating profit flow, given the imposition of a royalty obligation, and 2) the corresponding required rate of return to the licensor, given the riskiness of the royalty stream that the licensor will receive.

We then go on to explain how to estimate the three types of required rates of return to intangible assets. That is, we show the reader how to estimate the required rate of return to a firm's stream of anticipated residual profit, the required rate of return to a licensee's operating profit, and the required rate of return to a licensor's operating profit.

Understanding how to estimate discount rates for these three kinds of intangible capital returns is important for at least two reasons. First, it is difficult to reliably value or price intangible assets without a clear understanding of how to discount the income streams generated by those assets. For example, a common objective in valuation and licensing contexts is to discount a royalty obligation to present value, which under certain conditions is referred to as a "relief from royalty" method of valuing certain intangible assets. However, while the relief from royalty method should generally be implemented using a discount rate that appropriately reflects the systematic risk of the profit stream expected by the licensor in its licensing activity, in our experience the fact that the profit stream of the licensor might not have the same risk characteristics as, say, an operating profit / cash flow stream, is rarely considered.

Second, without a clear understanding of the *required* rate of return to intangible asset investments, we cannot determine whether *actual* or *realized* rates of return exceed the required rate of return. That is, we cannot tell whether the firm is earning any economic net residual profit.

It bears noting that while we necessarily discuss the nature of, and the formula for, the firm's weighted average cost of capital (WACC), we do not spend much time in this chapter (or for that matter in this book) on the topic of *how one should estimate* the weighted average cost of capital – despite the fact that the WACC is the starting point for our analysis of the three intangible asset-related discount rates. The reason is that WACC estimation is fairly well trod ground, and the reader can find comprehensive coverage of it elsewhere. Therefore, we discuss certain characteristics of the WACC at a high level, in order to provide the reader with a thorough understanding of how the cost of intangible capital can be derived, and how it relates to the firm's overall cost of capital (the WACC). Therefore, if necessary, the reader is encouraged to develop at least a basic understanding of the ideas behind the costs of equity and debt, and how to estimate these capital costs, on his or her own.[1]

II. Relating the Three Intangible Asset Discount Rates to the WACC

A. Decomposing the WACC – Overview

All three types of intangible asset discount rates that we cover here are derived from an estimate of the weighted average cost of capital, which is the return to the firm's entire asset portfolio, inclusive of intangible assets. Exhibit 3-1, below provides a graphical depiction of the most basic quantitative relationships between the WACC and the required rates of return covered here.

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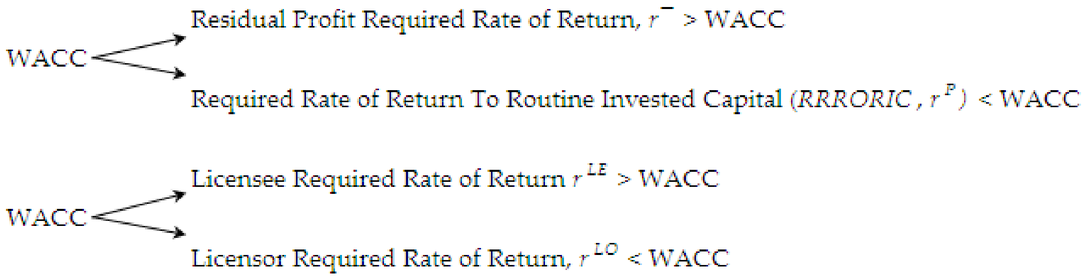
[1] See, for example, Pratt and Grabowski, Cost of Capital, 3rd Ed., (Wiley, 2008).

Exhibit 3-1
Relationship between the WACC and Intangible Asset Capital Costs



Beginning with the top panel of Exhibit 3.1, we start from the assumption that the WACC is the weighted average required rate of return to all of the firm's assets – that is, the firm's routine and non-routine, tangible and intangible capital.[2] This can be understood intuitively by simply looking at the accounting equation: A ≡ L + E, or Assets (*i.e.*, Capital) ≡ Liabilities + Equity. Because the WACC is the required return to the right hand side of this equation (it is the weighted average required return to the firm's liabilities, or debt financing, as well as its equity financing), it is also the required return to the left hand side of this equation (assets). This means that the WACC is the weighted average required rate of return to the firm's entire asset base.

If we then divide the firm's asset portfolio into two primary categories, routine assets and non-routine intangible assets, the former claiming a routine profit flow and the latter claiming residual profit, we see that the WACC can be decomposed into a required rate of return to routine profit, and a required rate of return to residual profit. The former we referred to in Chapter 2 as the rate of return to routine invested capital (RRRORIC), and the latter we referred to as the residual profit discount rate ($\bar{r}$).

The bottom panel of Exhibit 3.1 shows a similar decomposition of the WACC, this time into a required rate of return to "licensee profits," and a corresponding required rate of return to "licensor profits." Importantly, however, the bottom panel of Exhibit 3.1 is beginning from a slightly different conception of the WACC than is the top panel. That is, in order to break apart the WACC into a licensor and licensee component, the WACC that one begins with must be the

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[2] It is important to note at this point that, technically, in the presence of taxes, a firm with debt financing will have a different weighted average cost of capital than an identical firm (a firm with identical assets) that is financed by equity only. How one should treat the "tax shield" that comes from debt financing when decomposing the WACC is discussed later in this chapter.

WACC that would apply to a notional single firm performing both the licensor's activities (investing in, and licensing, intangible assets) and the licensee's activities (exploiting the licensed intangibles). Given this starting point (we discuss how to arrive at this WACC later in this chapter), we can then model the shift in risk that occurs between a licensor and licensee when they enter into a typical licensing arrangement. In general, a licensing arrangement that specifies that the licensee pays a royalty to the licensor increases the systematic (*i.e.*, compensable) risk borne by the licensee, and decreases the risk borne by the licensor. This is why Exhibit 3.1 shows the licensee's required return as greater than the WACC, and the corresponding licensor cost of capital as lower than the WACC.

B. Do We Start With the Pre-tax or After-tax WACC?

The weighted average cost of capital is, as the name implies, the weighted average of the costs of capital (required returns) for the firm's two primary sources of funding – debt and equity. While other types of "hybrid" funding, such as preferred stock, can also be incorporated into the WACC, and should be incorporated if these types of funding are used by the firm, for our purposes we can simply assume that the firm uses only the standard debt and equity forms of financing.

Importantly, there are *two* WACCs that analysts apply when discounting cash flows – the asset WACC (also known as the pre-tax WACC) and the after-tax WACC. Much confusion still exists in the valuation community around these two conceptions of the weighted average cost of capital.

In order to avoid this confusion, it is important to first understand why there are two WACCs, rather than one, used by economists and finance practitioners. The reason goes back to two classic papers published by Modigliani and Miller in 1958 and 1963, which produced the famous capital structure irrelevance theorem.

In brief, Modigliani-Miller showed that under perfect capital market conditions (zero market imperfections), and no taxes, the value of the firm is unaffected by changes in its capital structure. In other words, an all equity financed firm is equal in value to an identical firm (a firm with an identical portfolio of assets) that is financed by a combination of debt and equity. In such a case, because capital structure does not change the operations of a firm (therefore cash flows are unchanged), and because the firm's value is unchanged, it must be the case that the unlevered cost of equity (asset WACC) for an all equity firm is equal to the weighted average cost of equity and debt for a for a firm that is financed by both equity and debt. In other words, the increase in the cost of equity that is caused by introducing leverage exactly offsets the benefits to shareholders of using low cost (relative to equity) debt financing. The firm's *weighted average* cost of capital is unchanged when it moves from equity financing to a combination of equity and debt.

This, in turn, means that in perfect capital markets there is *only one WACC*. There is no real distinction between what we call the asset WACC (unlevered cost of equity) and the weighted average cost of debt and equity. The firm has one cost of capital.

However, market conditions are not perfect, and one critically important market distortion involves governments' provision of a significant subsidy to firms in the form of interest deductibility – also known as the interest "tax shield" (TS). From an economic perspective, the deductibility of interest distorts prices – by directly changing (lowering) the cost of debt capital. That is, the deductibility of interest means that the government subsidizes, or in essence pays for, txdxD of the firm's debt cost through lower tax receipts (where t is the income tax rate, d is the interest rate paid on debt,[3] and D is the amount of debt taken on by the firm). Correspondingly, the firm's shareholders now only have to pick up the remainder, or (1-t)xdxD of the total cost of debt.

This means that in the presence of interest deductibility, the firm's owners (the equity holders) are made better off by an amount roughly equal to (txdxD) / d, which is just txD.[4] This expression, txD, is known as the value of the "tax shield." <u>The tax shield is part of (included in) the firm's enterprise value and equity value</u>, but is not part of the firm's debt value (which remains equal to dxD / d, or just D).

So, how does all of this relate to the question of which WACC to use? As Ruback (1992) discusses in some detail, the tax shield (interest subsidy) means that the firm's so-called "capital cash flows" (CCF), are now higher than they would have

[3] Technically, the cost of debt at any given time is the firm's debt yield. However, for purposes of this discussion, we refer to the cost of debt as the debt interest cost.

[4] Note that the formula (txdxD) / d is just the formula for the present value of a perpetuity. This assumes implicitly that the amount of debt, D, is constant through time, and that the debt interest rate is equal to the debt cost of capital. These assumptions are examined carefully (and are relaxed) in the finance literature on capital structure, but most practitioners and theorists make these assumptions for simplicity.

been but for the subsidy. Capital cash flows are defined as the cash flow accruing to debt holders in each period (*d*x*D*) plus the cash flow accruing to equity holders, which is now *e*x*E* + *t*x*d*x*D*, where *e* is the cost of equity and *E* is the amount of the firm's equity financing. In other words, the firm's capital cash flows have risen because the tax shield is now added to the equity holders' returns.

Ruback demonstrates that in the presence of a tax shield on interest, there are two equivalent ways to properly arrive at the firm's enterprise value – or, put differently, two ways to compute enterprise cash flows and then discount these flows to present value. First, one can discount the firm's capital cash flows directly (again, the capital cash flows are the cash flows actually accruing to equity and debt holders, inclusive of the tax shield that benefits equity holders) at the firm's so-called "pre-tax WACC."

Also referred to as the "unlevered equity cost of capital" and the "asset WACC," the pre-tax WACC is just the WACC under Modigliani-Miller's perfect capital markets assumption. It is expressed as follows.[5]

(Formula 3-1) $$WACC^{PT} = d\left(\frac{D}{D+E}\right) + e\left(\frac{E}{D+E}\right).$$

Note that Formula 3-1 does *not* multiply *d* by (1-*t*), which is the standard way to account for the reduction in the firm's actual interest cost due to the debt interest shield. Ruback shows that the reason that one should not multiply *d* by (1-*t*) is that doing so lowers the WACC by an amount that results in a present value increase, or increment, that is *exactly equal to the value of the tax shield.* In other words, if one were to discount the CCF by $d(1-t)\left(\frac{D}{D+E}\right) + e\left(\frac{E}{D+E}\right)$, which is also known as the "after-tax WACC," one would effectively double count the tax shield. This is because, as we have shown, the capital cash flows already include the tax shield. Therefore, application of the (1-*t*) factor to *d*, which also results in

[5] It bears noting that the pre-tax WACC can also be derived using the standard CAPM (capital asset pricing model), so long as the beta coefficient used is a fully "unlevered" beta. As is well known, the CAPM says that the cost of equity capital is equal to r^f + *B* × *ERP*, where r^f is the risk free rate of return, *B* is the beta coefficient (a measure of the covariance of an asset's returns with a fully diversified portfolio of assets), and *ERP* is the equity risk premium. Given that an unlevered beta (a beta computed after adjusting for the effects of leverage on beta) measures the riskiness of the firm assuming it carries no debt capital, the unlevered CAPM must provide the weighted average required rate of return to all assets. Thus, the unlevered beta coefficient is often referred to as the "asset beta." This is exactly the same thing as the pre-tax WACC shown in formula 3-1.

an increase in the resulting present value figure that is equal to the tax shield value, effectively results in double count of the tax shield's value.

The second way to properly arrive at enterprise value in the presence of a tax shield on interest is to begin with a *notional*, or *fictitious*, cash flow stream that Ruback (and others) refer to as "free cash flow" (FCF), and then discount this at the after-tax WACC. Instead of including the tax shield, FCF is effectively equal to the cash flows that would accrue to equity and debt in the *absence* of a debt interest subsidy – *i.e.*, the capital cash flows that would occur if no tax shield was present, or CCF – TS. Free cash flow is equal to operating cash flow less what we call notional taxes (NT).

As noted directly above, the after-tax WACC that is applicable to FCF is equal to:

(Formula 3-2) $$WACC^{AT} = d(1-t)\left(\frac{D}{D+E}\right) + e\left(\frac{E}{D+E}\right).$$

Put differently, because $WACC^{AT}$, being lower than $WACC^{PT}$, produces a present value increment that is equal to the tax shield, one cannot include the tax shield in the cash flows that one discounts using $WACC^{AT}$ (otherwise one would, again, be double counting the TS).[6]

In summary, we have two definitions of cash flow and two discount rates that should be applied to our two cash flow definitions. First, we have capital cash flow, which is the actual cash flow that accrues to debt and equity holders, inclusive of the tax shield received by the equity holders. Capital cash flow is equal to operating profit less cash taxes. Capital cash flow should be discounted at the pre-tax WACC. Second, we have free cash flow, which is less than CCF by the amount of the tax shield. FCF equals operating profit less notional taxes, or the taxes that would be borne by the firm if the interest subsidy were not present. FCF should be discounted by the after-tax WACC.

Now that we have established that the presence of the tax shield implies that two versions of the WACC exist, corresponding to two calculations of the firm's cash flows, the question becomes: which one should we use as our starting point for decomposition into the various intangible capital-related returns shown in Exhibit 3-1? Which WACC should be used to discount operating profit?

[6] The technical appendix to this chapter includes a very straightforward flow chart, taken directly from Ruback (1992), that shows the process by which one derives FCF and CCF in the presence of a tax shield.

The answer follows directly from the discussion above. Given that our starting point for decomposition is a pre-tax measure (operating profit or operating cash flow), and that the present value of expected operating profit equals the value of all of the claims to the firm's operating profit, inclusive of the government's claim (*i.e.*, taxes), we must use a discount rate that, when applied to a forecast of the operating profit, produces a present value that is equal to the firm's enterprise value (the value of debt plus equity) plus the present value of corporate taxes. A starting discount rate that does not meet this criterion is, by definition, one that will overvalue or undervalue the firm's assets.

The discount rate that meets this criterion is the *pre-tax WACC*. By contrast, the after-tax WACC does not meet this criterion.

To see this, first assume for simplicity that the firm is in a steady state and has a zero growth rate. The zero growth rate assumption implies that present values can be taken using the perpetuity formula (*i.e.*, simply dividing the constant cash flow by the discount rate).

As shown in Formula 3-3 below, operating profit is equal to capital cash flows plus the firm's cash taxes.

(Formula 3-3) $$\pi = CCF + CT,$$

where CT is cash taxes. Translating formula 3-3 into present values we have:

(Formula 3-4) $$\frac{\pi}{WACC^{PT}} = \frac{CCF}{WACC^{PT}} + \frac{CT}{WACC^{PT}} = EV + PV(CT).$$

Formula 3-4 simply says that just as operating cash flow equals capital cash flows plus cash taxes, the present value of operating cash flow using the *pre-tax WACC as the discount rate* equals the firm's enterprise value (the market value of debt and equity) plus the present value of the government's claim over the cash flows generated by the firm's assets. Thus, the pre-tax discount rate meets the criterion.

As for whether or not the after-tax discount rate also meets the criterion, we begin in the same way as we did in Formula 3-3, but we examine free cash flow, which we know is discounted by the after-tax discount rate in order to arrive at enterprise value. Given that operating profit is equal to free cash flow plus notional taxes, we have formula 3-5, below.

(Formula 3-5) $$\pi = FCF + NT.$$

The corresponding present value equation would be:

(Formula 3-6) $$\frac{\pi}{WACC^{AT}} = \frac{FCF}{WACC^{AT}} + \frac{NT}{WACC^{AT}} = EV + PV(NT).$$

Inspection of formula 3-6 tells us immediately that discounting operating profit at the after-tax WACC does not meet the criterion that the resulting present value equals enterprise value plus the government's claim over the firm's assets (*i.e.*, the total amount of value in the assets of the firm). Recalling that FCF does not contain the tax shield, but that discounting at the after-tax WACC produces a present value increment equal to the tax shield, we know that the first term on the right hand side of formula 3-6 is in fact enterprise value. However, the second term on the right hand side is necessarily *greater than* the government's claim over the enterprise. This is true simply because notional taxes (NT) are greater than cash taxes, by the amount of the tax shield. This, in turn, means that discounting operating profit by the after-tax WACC double counts the tax shield contained within operating profit. In other words, the value of the tax is in both the first term on the right hand side of formula 3-6, and the second term. Therefore, discounting operating profit at the after-tax WACC overvalues the operating profit stream.

Thus, our conclusion. The decomposition of the WACC should begin with the pre-tax WACC, as defined in formula 3-1. Intuitively, this comports nicely with the idea, also taken from Modigliani Miller, that the pre-tax WACC is the *asset WACC*. That is, the pre-tax WACC is also the weighted average required rate of return to the firm's asset categories.

C. Two Approaches to the Decomposition of the Asset WACC

Once again, at a fundamental level, the $WACC^{PT}$ (which we will also refer to as *r*) can be thought of as a weighted average of the costs of capital for: 1) routine and non-routine profit flows, and 2) licensor and licensee profit flows. This means that if we know *r*, and we are decomposing it into two components (*e.g.*, routine and non-routine capital costs), then we must know the cost of capital for one of the components in order to solve for the other. In other words, if we know that *r* is the weighted average of *RRRORIC* and $\bar{r}$, then we must either know *RRRORIC* to obtain $\bar{r}$, or $\bar{r}$ to obtain *RRRORIC*. The same logic applies to decomposition of *r* into licensor and licensee capital costs.

There are two ways of getting at the problem of finding one of the "decomposition components" (one of the two the decomposed capital costs). These are: 1) direct empirical observation, and 2) model-based estimation. In other words, sometimes we can directly observe, say, the routine cost of capital. Given an estimate of *r*, direct empirical evidence of *RRRORIC* then allows us to solve for $\bar{r}$. On the other hand, if we cannot empirically observe either of the two components of $WACC^{PT}$, then we must use a financial model to estimate one (which in turn allows us to solve for the other).

As we shall see, model-based estimation of capital costs exploits the finance literature related operating leverage. This literature shows us how the priority of claims on revenue affects the cost of capital. Given that some capital sources have priority over others – for example, routine capital has priority over intangible capital – the models from this literature can be used to form an estimate of the capital cost associated with, say, non-routine invested capital (residual profit) given that routine capital has a priority claim over operating cash flows.

We begin in Section III by decomposing of $WACC^{PT}$ into routine and non-routine components. Section IV then applies very similar reasoning to the decomposition of $WACC^{PT}$ into licensor and licensee components.

III. Decomposing the WACC into Routine and Non-routine Components

A. Model-based Decomposition

1. Operating Leverage and Risk

Operating leverage has been studied extensively by finance practitioners, and accepted models exist for determining its effect on the required return to invested capital. These models generally rely on the CAPM framework (capital asset pricing model), and center the analysis on the effect of operating leverage on the "asset beta," described earlier in footnote 5.

The CAPM, which remains the workhorse cost of capital model in finance and economics, says that the required return on an asset is a function of the "systematic" risk of the cash flows generated by that asset. Systematic risk is risk that cannot be diversified away.

The CAPM derives a single measure of systematic risk, represented as the variable *B* (beta), that flows into the following simple formula:

(Formula 3-7) $R = r^f + (B \times ERP) = e,$

where *R* and *e* are the required rate of return (in percentage points), r^f is the risk free rate of return, *B* is the beta coefficient, and *ERP* is the equity risk premium (the average long run return earned in the stock market less the risk free rate). Obviously, the higher is Beta, the higher is the required rate of return.

Each company (and in fact each kind of asset) has its own beta. Betas for publicly traded companies are easy to derive, and are easy to find in the public domain.[7]

How does operating leverage influence Beta? The most cited treatment in finance on this issue is Mandelker and Rhee (1984), who define a measure called the "degree of operating leverage," or "DOL."

DOL is really just the elasticity of operating profit with respect to changes in sales. The more sensitive, in percentage terms, is operating profit to changes in sales, the higher is the degree of operating leverage. This makes sense, since firms with high fixed costs relative to variable costs will generally see larger movements in operating profit when sales spike upward or downward.

Formally, the DOL is defined as follows:

(Formula 3-8) $DOL = \frac{\%\Delta(\text{Operating Profit})}{\%\Delta(\text{Revenue})}.$

Mandelker and Rhee show that a company's operating asset beta, which is the beta coefficient for the entire enterprise, can be decomposed as follows.

(Formula 3-9) $B_j = (\text{DOL})^* B_j^0,$

[7] See Yahoo! Finance, for example.

Where B_j is the beta for company *j*, and B_j^0 is the company's "intrinsic systematic risk." Intrinsic systematic risk is the systematic risk of the company's operating assets, assuming no operating leverage (that is, assuming that all costs are variable).

Formula 3-9 is both very simple, and very general. Mandelker and Rhee's formula can be used to compare different operating leverage positions, and their effect on beta, for a company. In other words, the formula can be used to examine the impact on beta of *increasing* the operating leverage of a company – or a specific cash flow stream.

Formally, this is expressed as:

(Formula 3-10) $\%\Delta B_l = (\%\Delta DOL) * B_l^0$,

Where $\%\Delta B_l$ is the incremental increase in beta given the increase in operating leverage, and B_l^0 is the beta at the base (pre-increase) level of operating leverage. This formula provides us with the starting point for our analysis. That is, we can use this formula as the starting point for deriving the cost of capital for residual profit, given that residual profit faces higher operating leverage than total operating profit due to the priority of the routine capital's claim over operating profit. Moreover, we can also exploit Formula 3-10 to examine the change in a licensor's or licensee's cost of capital as a function of changes in a royalty rate applied to that company.

2. *Using DOL to Estimate $\bar{r}$*

There are two primary ways to model the way in which the priority claim of routine profit flows affect the degree of operating leverage faced by non-routine assets in a steady state.[8] First, we can assume that routine invested capital is fixed in proportion to total invested capital, and therefore the flow of operating profit claimed by routine invested capital is also fixed in proportion. Under this assumption, the claim of routine invested capital in each period is simply $\pi^P = RRORIC = RIC \times (r^P - g)$, as given in formula 2-4.

[8] Both of these involve simplifying assumptions that can be relaxed to account for more complex fact patterns.

Alternatively, we can assume that routine invested capital claims a constant operating profit margin or markup on total costs (these are equivalent). Here, we assume that routine profits equal total cost times *m*, which is a markup factor.

Each of these two modeling approaches is developed below.

a) Fixed RIC Proportion

Assuming that *RIC* represents a fixed percentage of total invested capital, the effect on the riskiness of residual profit from the *RIC*'s priority claim over cash flows can be modeled as follows. First, we can write out the formula for net residual profit, given the priority claim of the routine invested capital, as follows.

(Formula 3-11) $\pi^{RN} = S(1 - v) - C - \pi^P,$

Where π^{RN} is net residual profit, *S* is sales, *v* represents variable costs as a percentage of sales, *C* is fixed costs, and π^P is defined as before (π^P equals *RRORIC*).

Given that the *DOL* is defined as the elasticity of operating profit with respect to sales. The elasticity of any variable *Y*, with respect to another variable *X* is always defined as $\frac{\frac{\partial Y}{\partial X}}{\frac{Y}{X}}$. In this case, then, the elasticity of interest is the elasticity of net residual profit with respect to changes in revenue, or $\frac{\frac{\partial \pi^{RN}}{\partial S}}{\frac{\pi^{RN}}{S}}$. This can be expressed as follows.

(Formula 3-12) $DOL^{\pi^{RN}} = \frac{\frac{1-v}{1}}{\frac{S(1-v)-C-\pi^P}{S}} = \frac{S(1-v)}{S(1-v)-C-\pi^P},$

where $DOL^{\pi^{RN}}$ is the degree of operating leverage of net residual profit.

From here, it is a short step to determine the adjustment to beta that results from the imposition of *RIC*'s priority claim over operating profit. The adjustment is simply *DOL* given *RIC*'s priority claim (*i.e.*, the DOL that we just derived, the DOL for net residual profit), divided by *DOL* for all operating profit.

The DOL for all operating profit is derived exactly the same way as the DOL for net residual profit. That is, we derive the elasticity of operating profit with respect to changes in revenue, or $\frac{\frac{\partial\pi}{\partial S}}{\frac{\pi}{S}}$. This can be expressed as follows.

(Formula 3-13) $$DOL^{\pi} = \frac{\frac{1-v}{1}}{\frac{S(1-v)-C}{S}} = \frac{S(1-v)}{S(1-v)-C},$$

where DOL^{π} is the degree of operating leverage of total operating profit.

Our beta adjustment is then simply:

(Formula 3-14) $$Beta\ Adjustment = \frac{DOL^{\pi^{RN}}}{DOL^{\pi}} = \frac{\frac{S(1-v)}{S(1-v)-C-\pi^{P}}}{\frac{S(1-v)}{S(1-v)-C}} = \frac{S(1-v)-C}{S(1-v)-C-\pi^{P}} = \frac{\pi}{\pi^{RN}}.$$

Notice that the beta adjustment that accounts for the operating leverage faced by net residual profit (non-routine intangible capital) is <u>simply the formula for net residual profit divided by the formula for operating profit</u>, or $\frac{\pi}{\pi^{RN}}$. This means that the cost of capital for net residual profit can be written as:

(Formula 3-15) $$WACC^{\pi^{RN}} = r^{f} + B\left(\frac{\pi}{\pi^{RN}}\right)ERP,$$

Where B is the unlevered, or asset, beta.

b) Fixed Markup on Total Cost

For the case in which routine returns are determined using a constant markup on total cost, the math is slightly different. However, the process is similar. First, we compute the DOL for net residual profit. Second, we compute the DOL for operating profit. Finally, we derive the beta adjustment by dividing the DOL for net residual profit by the DOL for operating profit.

It bears noting that the use of a total cost plus markup to remunerate routine invested capital is very common in transfer pricing exercises. Frequently, large multinational corporations will ensure that their transfer pricing structures remunerate "routine" entities (*e.g.*, internal contract manufacturers) using a fixed markup on total cost.

In this case, the net residual profit function can be written as:

(Formula 3-16) $\pi^{RN} = S[1 - v(1 + m)] - C(1 + m)$,

where m is the markup on total cost. Now, the elasticity formula is derived exactly as before, and the result is as follows.

(Formula 3-17) $$DOL^{\pi^{RN}} = \frac{\frac{\partial \pi^{RN}}{\partial S}}{\frac{\pi^{RN}}{S}} = \frac{S[1-v(1+m)]}{S[1-v(1+m)]-C(1+m)}.$$

We already know DOL^{π} from formula 3-13. This means that we have the two components necessary to form our estimate of the beta adjustment. The adjustment is given in formula 3-18, below.

(Formula 3-18) $$Beta\ Adjustment = \frac{DOL^{\pi^{RN}}}{DOL^{\pi}} = \frac{\frac{S[1-v(1+m)]}{S[1-v(1+m)]-C(1+m)}}{\frac{S(1-v)}{S(1-v)-C}} = \frac{S[1-v(1+m)][S(1-v)-C]}{[S[1-v(1+m)]-C(1+m)][S(1-v)]}.$$

This, finally, implies that the cost of capital applicable to net residual profit, the case wherein the return to routine invested capital is computed as a markup on total cost, is:

(Formula 3-19) $$WACC^{\pi^{RN}} = r^f + B\left(\frac{S[1-v(1+m)][S(1-v)-C]}{[S[1-v(1+m)]-C(1+m)][S(1-v)]}\right) ERP.$$

c) Some Properties of $\bar{r}$ Given Model-based Estimation

Exhibits 3-2 and 3-3, below, show the application of our model-based approach to estimating the cost of capital for the firm's residual profit flow. Both exhibits assume that *RRORIC* is computed as a markup on total cost. The analogous calculations, assuming that *RRORIC* is a fixed return on routine invested capital that is assumed to be a fixed proportion of total capital, are left to the reader as an exercise.

Exhibit 3-2

Estimation of Cost of Non-routine Intangible Capital
Routine Return Modeled as Fixed Markup on Total Cost

Line	Variable	Value	Calculation
1	*S*	100	Assumed
2	*v*	45.0%	Assumed
3	*C*	40	Assumed
4	*m*	7.0%	Assumed
5	r^f	4.0%	Assumed
6	*B*	1.2	Assumed
7	*ERP*	6.0%	Assumed
8	$WACC^{PT}$	11.2%	Line 5 + Line 6 * Line 7
9	Beta Adjustment	1.56	Application of Formula 3-18
10	Non-routine Capital Cost	15.3%	Line 5 + Line 6 * Line 9 * Line 7
11	Assumed g	3.0%	Assumed
12	Operating profit	15	Line 1 * (1 - Line 2 * (1 + Line 4)) - Line 3 * (1 + Line 4)
13	Residual profit	9.0	Line 12 - (Line 1 * Line 2 * Line 4) - (Line 3 * Line 4)
14	Total Enterprise Value	182.9	Line 12 / (Line 8 - Line 11)
15	Present Value Non-routine Profits	73.9	Line 13 / (Line 10 - Line 11)
16	Routine capital cost	8.5%	[(Line 12 - Line 13) / (Line 14 - Line 15)] + Line 11
17	Present Value Routine Profits	109.1	(Line 12 - Line 13) / (Line 16 - Line 11)

As shown in exhibit 3-2, under fairly realistic assumptions, the adjusted beta is a full 56 percent higher than the beta for the firm as a whole (line 9), and the cost of capital is a full 4.1 percentage points higher. Exhibit 3-3, directly below, offers a sensitivity analysis.

Exhibit 3-3

Sensitivity Analysis
Non-routine and Routine Capital Costs as a Function of Markup Rate
Given Other Parameters Shown in Exhibit 3-2

Markup Rate	$WACC^{PT}$	Non-routine Capital Cost	Routine Capital Cost
5%	11.2%	13.64%	8.2%
6%	11.2%	14.37%	8.3%
7%	11.2%	15.25%	8.5%
8%	11.2%	16.31%	8.6%
9%	11.2%	17.61%	8.8%
10%	11.2%	19.26%	8.9%

B. Direct Observation-based Decomposition

Suppose that we have a firm whose enterprise value is equal to:

(Formula 3-20) $$TEV = \int_0^\infty \pi e^{-rt} e^{gt} dt = \int_0^\infty \pi e^{-(r-g)t} dt,$$

where *TEV* is the firm's *total* enterprise value, which is enterprise value plus the government's claim (cash taxes), *r* is the asset WACC (pre-tax WACC), *g* is the firm's instantaneous rate of growth per annum, and π is the firm's operating cash flow. In short, formula 3-20 is simply the present value of the firm's operating profit flow.

As shown in Appendix II-A, a little manipulation of formula 3-20 results in the following, surprisingly simple, formula for total enterprise value:

(Formula 3-21) $$TEV = \frac{\pi}{r-g}.$$

Formula 3-21 should be familiar to readers with a background in finance or economics – it is simply the present value formula for a continuous flow of operating profit, growing at a constant rate. Formula 3-21 was used in Exhibit 3-2, earlier.

How, then, can we decompose r into its routine and non-routine components without the aid of the operating leverage models developed above?

First, let's denote the routine discount rate (RRRORIC) as r^P. We then use $\bar{r}$ for the residual profit discount rate. We now decompose the firm's total enterprise value into its routine invested capital component, and its residual profit or non-routine intangible capital component. This is given in Formula 3-22.

(Formula 3-22) $$TEV = \frac{\pi}{r-g} = \frac{RRORIC}{r^P-g} + \frac{\pi^{RN}}{\bar{r}-g}.$$

Formula 3-22 just says that the firm's intrinsic enterprise value must equal the present value its required return to routine invested capital, plus the present value of the firm's net residual profit. Formula 3-22 can be easily manipulated in order to solve for the residual profit discount rate, $\bar{r}$.

(Formula 3-23) $$\bar{r} = \frac{\pi^{RN}}{\left(\frac{\pi}{r-g} - \frac{RRORIC}{r^P-g}\right)} + g = \frac{\pi^{RN}}{(TEV-RTEV)} + g,$$

where *RTEV* is the present value of the operating cash flows generated by routine invested capital.[9] Put differently, *RTEV* is the "routine" *TEV*, or as Warren Buffett would call it, the "business" value, as distinct from the "franchise" value.

Formula 3-23 gives us $\bar{r}$ as a function of operating profit, residual profit, routine profit, the asset WACC, the expected growth rate of the firm, <u>and the routine discount rate</u>. This means that we can <u>solve for</u> the firm's $\bar{r}$, given an empirical estimate of the routine discount rate. As discussed in Chapter 2, this estimate can be obtained by directly observing the cost of capital for benchmark companies operating in the same, or a similar, industry, but operating without the aid of the non-routine invested capital that is owned by the firm that we are studying.

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[9] Notice from the right hand side of formula 3-23 that $\bar{r}$ is equal to the ratio of net residual profit to the present value of net residual profit, plus g. This is because the denominator of the first term on the right hand side of the equality is, by definition, the present value of net residual profit.

C. Simplified Example, and Some Properties of $\bar{r}$

Formula 3-23 shows us that the residual profit discount rate is a function of just five variables: operating profit, *RRORIC*, r, r^P, and g. Exhibit 3.4, below, provides an example calculation of $\bar{r}$.

Exhibit 3-4

Example Calculation
Required Rate of Return to Residual Profit Stream (Required Return to IDCs)

Line Item	Value	Calculation
Firm Financial Variables		
1 Sales	$100	Given
2 Operating Margin	20%	Given
3 Operating Profit	$20	Line 1 x Line 2
4 Routine Operating Margin	12%	Given
5 *RRORIC*	$12	Line 1 x Line 4
6 Residual Profit	$8	Line 3 - Line 5
Rate of Return and Growth Variables		
7 r^f	5%	Given
8 *B*	1	Given
9 *E*	6%	Given
10 Pre-Tax WACC	11.00%	Line 7 + Line 8 x Line 9
11 *RRRORIC*	9%	Given
12 g	3%	Given
Intermediate Variables		
13 *TEV*	$250	Line 3 / (Line 10 - Line 12)
14 *RTEV*	$200	Line 6 / (Line 11 - Line 12)
Result		
13 $\bar{r}$	19.0%	Line 6 / (Line 13 - Line 14) + 12

Exhibit 3.4 displays the calculation of $\bar{r}$ in four sections. The top two sections are input variables, including the firm's assumed P&L results, routine profit assumptions, and assumptions regarding the pre-tax WACC. Notice that the pre-tax WACC is developed using the standard capital asset pricing model (CAPM) with no adjustments to the beta coefficient for leverage, as per footnote 5 above. The third section calculates the two terms in the denominator of the right hand side of formula 3-23. Finally, the fourth section gives us our result of 19 percent. It should be noted that at 19 percent, the residual profit discount is obviously quite a bit higher than the pre-tax WACC. This is a typical result. In general, we find that the cost of capital is in the upper teens or low 20 percent range.

Visual inspection of formula 3-23 reveals several important relationships that should hold among the firm's residual and routine profit flows, and their respective discount rates. For example, it should be clear that the firm must have an *RTEV* that is less than its *TEV* – otherwise, the denominator on the right hand side of formula 3-23 will be less than or equal to zero. Given that *TEV* and *RTEV* are equal to $\frac{\pi}{r-g}$ and $\frac{RRORIC}{r^P-g}$, respectively, this implies the following.

(Formula 3-24) $$\frac{\pi}{r-g} \geq \frac{RRORIC}{r^P-g} \Rightarrow \frac{r^P-g}{r-g} \geq \frac{RRORIC}{\pi}.$$

Formula 3-24 says that the *net* routine discount rate (net of g) as a percentage of the net pre-tax WACC should be greater than routine profit as a percentage of total operating profit.

Thus, at a zero growth rate, that if the firm's *RRORIC* is equal to, say, 80 percent of its operating profit (π), then the required rate of return on routine invested capital, r^P, must be greater than 80 percent of the firm's pre-tax WACC. Otherwise, *RTEV* will be greater than or equal to *TEV*, which would mean that there is no value in the firm's residual profit stream.

The higher is g, the more rapidly the firm's net discount rate ratio $\frac{r^P-g}{r-g}$ approaches its $\frac{RRORIC}{\pi}$ ratio as r^P increases. In other words, while we stated above that at a zero growth rate, the threshold for r^P as a percentage of the pre-tax WACC is simply the firm's *RRORIC* as a percentage of π, with a positive expected growth rate the threshold is higher – meaning that *rP* must be even greater than *RRORIC* as a percentage of operating profit in order for the firm's residual profit stream to have any value.

Exhibit 3.5 shows the way in which the net discount rate ratio, the ratio of *RTEV* to *TEV*, and $\bar{r}$ behave as *rP* approaches *r* – for a given configuration of $WACC^{PT}$, *RRORIC*, π, and *g*.

Exhibit 3-5
Behavior of $\bar{r}$, *RTEV/TEV*, and the Net Discount Rate Ratio
As r^P Approaches *r*

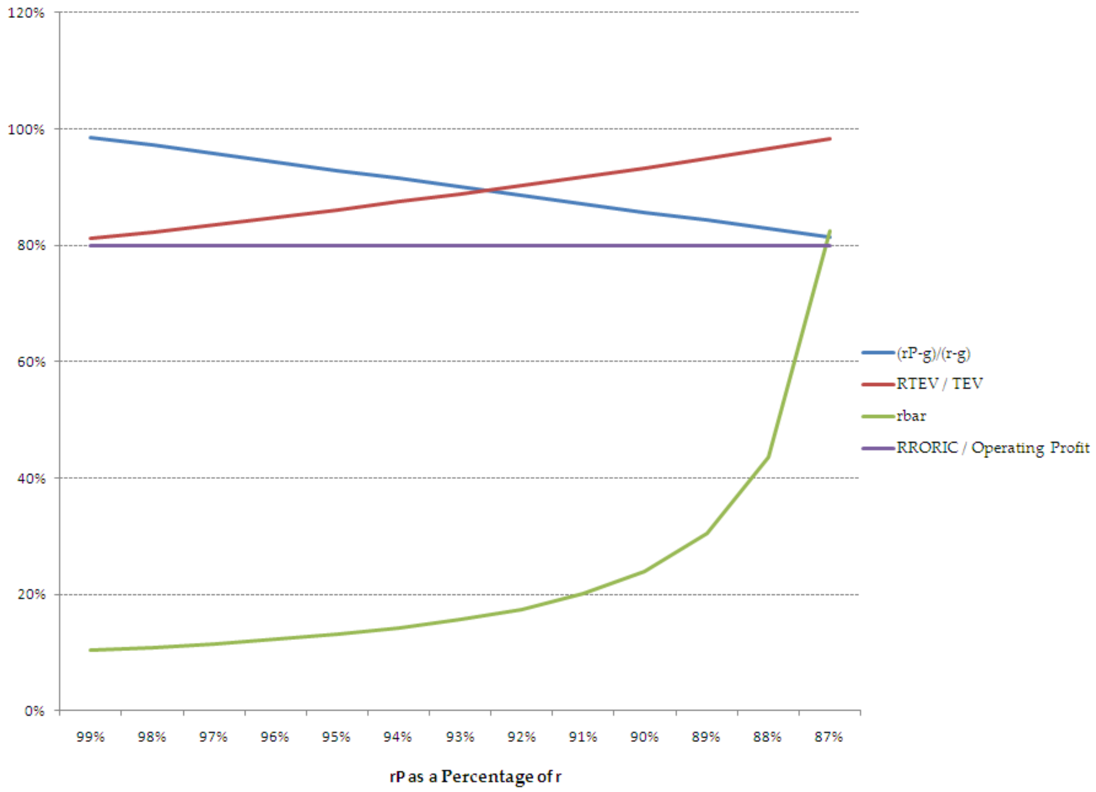


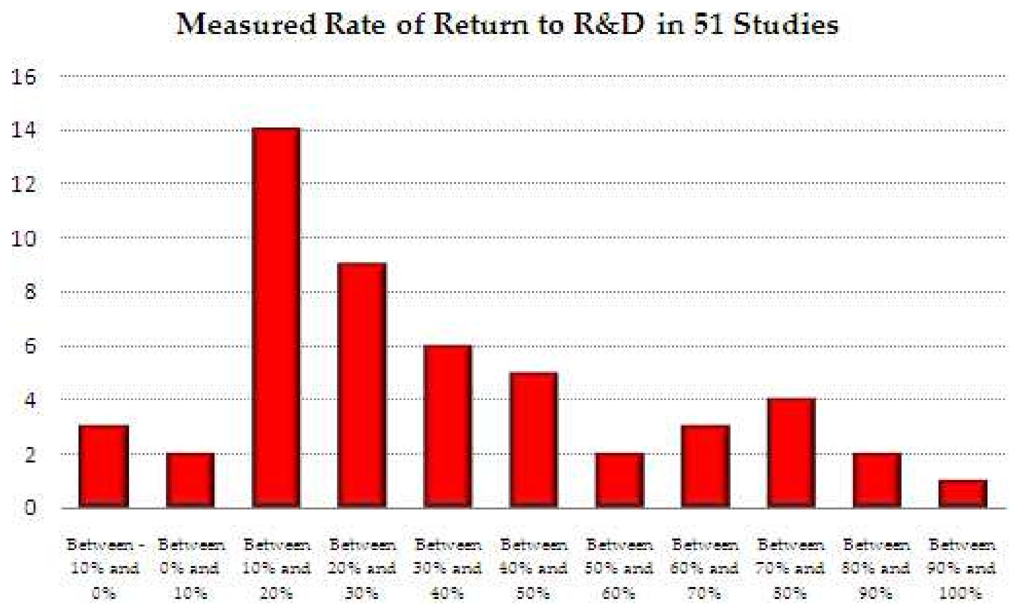
Exhibit 3.5 shows that, all else equal, as r^P decreases, *RTEV* rises toward *TEV* and the net discount rate ratio, $\frac{r^P-g}{r-g}$, falls toward the ratio of *RRORIC* to total operating profit. At the point where *RTEV* and *TEV* are equal the net discount rate ratio equals *RRORIC* as a percentage of operating profit, and $\bar{r}$ approaches infinity.

The point made in Exhibit 3.5 is this. The analyst should be careful that his or her estimate of r^P is high enough (close enough to the firm's pre-tax WACC) so that absurd conclusions are not reached for $\bar{r}$.

While it is clear that an $\bar{r}$ approaching infinity must be incorrect, what is less clear is where, exactly, should the analyst generally expect $\bar{r}$ to land? Some guidance is afforded us by the economics literatures on the rate of return to certain types of IDCs.

For example, Hall (2009) provides a comprehensive survey of the results of numerous economic studies that measure the rate of return to R&D. The exhibit below summarizes the findings of 51 studies surveyed by Hall.

Exhibit 3-6



The exhibit also shows a wide dispersion in the measured rate of return to R&D. This is caused by variation in time periods covered, industries studied, and econometric models used.

However, the exhibit also shows that the vast majority of the studies find that the rate of return to R&D is between 10 percent and 50 percent – with 34 of the 51 studies surveyed falling within this range. It is also clear that almost none of the studies found negative or single digit rates of return, and only 12 of the 51 studies found rates of return above 50 percent.

The median study found that the rate of return to R&D was 29 percent. The average from the studies surveyed was 31 percent. According to Hall, these represent reasonable estimates of the rate of return to R&D.

The exhibit below provides a summary of the studies surveyed by Hall, and their rate of return findings. Summary statistics are given at the bottom of the exhibit, including the interquartile range.

Exhibit 3-7

R&D Rates of Return Across 51 Studies

Study	Rate of Return
Mairesse-Cuneo (1985)	-128%
Cuneo-Mairesse (1984)	-90%
Mairesse-Cuneo (1985)	0%
Bernstein-Nadiri (1989)	7%
Mohnen(1992b)	8%
Klette (1991)	11%
Griffith-Harrison-van Reenen (2006)	11%
Bernstein (1988)	12%
Mohnen-Nadiri-Prucha (1986)	13%
Griffith-Harrison-van Reenen (2006)	14%
Nadiri-Kim (1996a)	14%
Bernstein and Nadiri (1990)	15%
Bernstein (1998)	15%
Nadiri-Kim (1996b)	15%
Hall-Mairesse (1995)	16%
Bernstein-Yan (1997)	17%
Odagiri-Iwata (1986	19%
Clark-Griliches (1984)	19%
Mohnen (1990)	20%
Wang-Tsai (2003)	22%
Mairesse-Mohnen-Kremp (2005)	22%
Hall (1993)	22%
Verspagen (1995)	23%
Hall-Foray-Mairesse (2009)	23%
Hall-Mairesse (1995)	27%
Bond-Harhoff-van Reenen (2005)	29%
Lichtenberg and Siegel (1991)	29%
Wakelin (2001)	29%
Hall (1993)	31%
Bond-Harhoff-van Reenen (2003)	32%
Medda-Piga-Siegel (2003)	33%
Griliches-Mairesse (1984)	35%
Ortega-Arigiles et al. (2009)	35%
Bernstein (1989)	36%
Bernstein-Mohnen (1998)	46%
Harhoff (1998)	48%
Griliches-Mairesse (1990)	49%
Rogers (2009)	49%
Rogers (2009)	49%
Mohnen-Lepine (1991)	56%
Griffith-Redding-van Reenen (2004)	57%
Griliches (1986)	64%
Mairesse-Hall (1994)	64%
Griliches-Mairesse (1984)	64%
Harhoff (1998)	71%
Sveikauskas (2000)	73%
Harhoff (1998)	76%
Hall-Mairesse (1995)	78%
Rogers (2009)	81%
Rogers (2009)	81%
Mairesse-Hall (1994)	92%
Bartelsman et al (1996)	102%
Lower Quartile	15%
Median	29%
Upper Quartile	51%
Mean	31%

While it is true that studies measuring the *actual* rate of return to R&D are not measuring the *required rate of return* to R&D, we should expect that many (if not most) of the firms sampled operate under monopolistic competition. Therefore, on average, these firms should experience actual rates of return to R&D approximately equal to their required rates of return.

Similar studies exist, arriving at similar rate of return results, for the actual rate of return to marketing and customer-based IDCs. For example, the work of Ayanian (1983), Hirschey (1982), and Graham and Frankenberger (2000).

IV. Decomposition into Licensor and Licensee Required Rates of Return

A. Decomposition of What?

Just as the firm's weighted average cost of capital can be decomposed into required rates of return for its routine and residual profit streams, in the same way we can view licensor and licensee capital costs as the result of a decomposition of a weighted average capital cost. Of course, licensors and licensees are typically not part of the same firm, so the first question that we must ask is: what discount rate, exactly, are we decomposing into a licensor and licensee cost of capital?

In related party contexts – such as related party licensing of intangibles – the answer is easy. That is, we have a related licensor and a related licensee that are in fact part of the same firm. Therefore, we can begin our analysis using the firm's WACC, and then decompose it into licensor and licensee capital costs.

Unfortunately, in unrelated party contexts, such as IP damages claims involving two parties that are adverse to one another, we don't have a single firm, with a single WACC. Therefore the question arises: which *notional* WACC are we decomposing? At a more practical level, how can we find comparable benchmark companies that both develop (the licensor function) and use (the licensee function) the intangibles being licensed? That is, if we can identify firms that perform both licensor intangible asset development functions, and licensee intangible asset exploitation functions, for the same or similar intangibles to those under study, then we have a starting point from which to work.

Typically, it is possible to find firms in the same industry as the licensor and licensee, that are comparable in their functions to the licensor and licensee "combined firm." For example, assume that we are interested in determining the proper discount rate to apply to a stream of royalties from a licensed pharmaceutical patent. A reasonable place to start our analysis is a sample of pre-tax WACCs for firms operating in the pharmaceutical industry that develop and exploit pharmaceutical technology similar to the patent at issue.

B. Royalty Obligations and Operating Leverage

Once we have a pre-tax WACC for our notional "combined firm" (or in the case of a related party license in a transfer pricing context, our actually combined firm), the next step is to model the way in which specialization affects the risk of the two parties to the firm. That is, we need to examine how the risk that is reflected in the pre-tax WACC is distributed between two parties – one of which specializes in technology development and out-licensing, and the other in in-licensing and commercial exploitation. The way to approach this problem, in our view, is to focus on the way in which a licensing arrangement affects the operating leverage of the two parties.

A royalty is obviously an operating cost. Intangible asset development such as R&D is also, obviously, an operating cost. The higher is the royalty rate paid by a licensee, or the higher are IDCs relative to revenue, the higher are the licensee's and licensor's operating costs relative to revenue. One can therefore think of an increase in a royalty rate as increasing a licensee's operating leverage, and a rising IDC-to-Sales ratio as doing the same to the licensor.

The corollary to this is that one can think of a royalty rate as a mechanism that not only transfers a share of revenue and profit *from the licensee* to the licensor, but that also transfers risk *to the licensee* from the licensor. This is so because, all else equal, increasing the royalty rate simultaneously increases the licensee's operating leverage while decreasing the licensor's operating leverage (the royalty increases the licensor's sales, thus decreasing the licensor's fixed IDC costs relative to sales).

In the sections that follow, we develop a simple model that captures the way in which the combined licensor + licensee pre-tax WACC is "distributed" between the licensor and licensee as the royalty rate changes.

C. The Licensee Side Mathematics

1. How Does Licensing Affect Operating Leverage?

From the perspective of the licensee, entering into a licensing arrangement has two effects. First, the licensee "offloads," or is relieved of, the fixed cost burden of certain intangible development costs. In the case of a technology license, for example, the licensee is relieved of the burden of bearing the R&D associated with the in-licensed technology. This offload of R&D represents a *decrease* in the licensee's operating leverage, and therefore cost of capital.

The reader may be wondering whether or not this fixed cost relief is really relevant. In other words, it is worth asking why IDC relief is in fact relevant to the licensee, since the licensee by definition *never had any IDC obligations*. Therefore, in what sense is he or she relieved of an IDC burden? The answer goes back to Section IV.A. of this chapter. That is, since the WACC that we are decomposing will usually be the WACC for firms that both develop and exploit IDCs related to the intangible assets of interest, in order to adjust the WACC for these benchmark firms to the licensee's operating leverage position, <u>two</u> changes must be accounted for: 1) the "offload" of fixed costs, and 2) the taking on of a royalty obligation.

Thus, the second adjustment that must be modeled when a licensee takes on a royalty obligation is the royalty obligation itself – which is a variable cost.[10] As it turns out, this *increases* the licensee's *DOL*.

The idea that a licensee's royalty obligation increases its operating leverage may at first seem counter-intuitive, because most license obligations involve a royalty rate applied as a percentage to sales. In other words, the royalty rate is a *variable cost* to the licensee, rather than a fixed cost. However, the reason that a royalty rate applied to sales increases the degree of operating leverage is very simple. A royalty obligation lowers the licensee's operating profit, which means that changes in sales will increase operating profit from a lower initial level – causing *larger percentage changes in operating profit*. Said differently, all else equal, the higher is a licensee's royalty rate, the larger will be the *percentage increase* in operating profit for a given percentage increase in the licensee's sales.

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[10] Note that we are assuming for purposes of this discussion that the licensee's royalty obligation is determined by the licensing contract to be a royalty rate, expressed as a percentage of sales, times sales.

The following table provides a simple numeric illustration of the way in which an increase in the licensee's royalty rate increases the *DOL*, all else equal.

Exhibit 3-8

Example Of Increasing DOL Through Increases In Royalty Rate

Line	Item	Firm A	Firm B	Calculation
A	Sales(t-1)	100	100	Given
B	Sales(t)	110	110	Assumed 10 percent increase in sales
C	Royalty Rate	0.0%	10.0%	Assumed Increase From 0% to 10% Royalty Rate
D	Fixed Costs (SG&A)	40	40	Assumed
E	Variable Costs (COGS)	40	40	Assumed
F	Operating Profit(t-1)	20	10	A*(1-C)-D-E
G	Operating Profit(t)	26	15	B*(1-C)-D-E
H	% Change Operating Profit	30%	50%	G/F - 1
I	% Change Sales	10%	10%	B/A - 1
J	DOL	300%	500%	H/I
K	% Change DOL		67%	J(Firm B) / J(Firm A)
L	% Change Beta		67%	K

The table compares two firms (Firm A and Firm B) that are identical in every respect, except that firm B has taken on a royalty obligation of 10 percent of sales, whereas firm A has no such royalty obligation. Both firms begin in period (t-1) with sales of 100, and both see a revenue increase in period (t) to 110. Similarly, both firms have fixed costs of 40, and variable costs of 40. Obviously, because of the 10 percent royalty obligation borne by firm B, its operating profit is lower in both periods (t-1) and (t), by 10 percent of sales. As a result of this, firm B's operating profit is more "elastic," meaning that it is more sensitive in percentage terms to sales increases, than is Firm A's operating profit.

As the table shows, Firm B's DOL is 67 percent higher than firm A's. This implies that Firm B's asset beta (and therefore systematic risk) is 67 percent higher than Firm A's level of risk.

2. *Formal Model*

a) *Effect of IDC Relief*

The effect on *DOL* of relief from the fixed cost burden related to ongoing investments in IDCs can be modeled as follows. First, we can write out the licensee's profit function, given investments in IDCs, as follows.

(Formula 3-25) $\pi^{LE} = S(1 - v) - C - I,$

where certain of our variables are defined as before (*S* is sales, *I* is intangible development costs), and the new variables introduced are *v*, which is the licensee's variable costs as a percentage of sales, and *C* which represents the licensee's fixed costs.

We noted that the *DOL* is defined as the elasticity of the licensee's operating profit with respect to sales. The elasticity of any variable *Y*, with respect to another variable *X* is always defined as $\frac{\frac{\partial Y}{\partial X}}{\frac{Y}{X}}$. In this case, then, the elasticity of interest is $\frac{\frac{\partial \pi^{LE}}{\partial S}}{\frac{\pi^{LE}}{S}}$. This can be expressed as follows.

(Formula 3-26) $DOL^{Relief} = \frac{\frac{1-v}{1}}{\frac{S(1-v)-C-I}{S}} = \frac{S(1-v)}{S(1-v)-C-I},$

where DOL^{Relief} stands for the degree of operating leverage given IDC relief. From here, it is a short step to determine the adjustment to beta from the IDC relief. The adjustment is simply *DOL* given relief, divided by *DOL* before relief. This is given in Formula 3-27, below.

(Formula 3-27) $IDC\ Beta\ Adjustment = \frac{DOL^{Relief}}{DOL^{No\ Relief}} = \frac{\frac{S(1-v)}{S(1-v)-C}}{\frac{S(1-v)}{S(1-v)-C-I}} = \frac{S(1-v)-C-I}{S(1-v)-C}.$

Notice that the beta adjustment for the change in operating leverage associated with the licensee's IDC relief is simply the licensee's profit function given no relief, divided by the profit function given relief.

b) Effect of Royalty Obligation

Now that we have adjusted the betas for our notional combined firms (*i.e.*, our comparable benchmark companies that both develop and exploit the intangibles) for the decrease in operating leverage, we in essence have the beta for a licensee that is paying a zero royalty rate. Of course, in point of fact, we know that the royalty rate that the licensee is paying is not zero. The question now is how can we adjust the "zero royalty" licensee beta to account for the imposition of a positive royalty rate.

A very similar approach to that taken above can be taken here. Given that the DOL is the percentage change in operating profit divided by the percentage change in revenues, we need to start with an expression for the licensee's operating profit.

(Formula 3-28) $$\pi^{LE} = S(1 - v - \Delta) - C,$$

Where Δ stands for the royalty rate, as a percentage of sales, that is imposed on the licensee. Notice that formula 3-28 has the licensee bearing no IDCs (I), but paying a royalty equal to ΔS.

Since DOL is the elasticity of operating profit with respect to changes in revenue, it is equal to the first derivative of π^{LE} with respect to S, or ($\partial\Pi / \partial S$), divided by the expression (Π / S). This is given below.

(Formula 3-29) $$DOL^{Royalty} = \frac{\frac{(1-v-\Delta)}{1}}{\left(\frac{S(1-v-\Delta)-C}{S}\right)} = \frac{S(1-v-\Delta)}{S(1-v-\Delta)-C}.$$

Where $DOL^{Royalty}$ is the degree of operating leverage experienced by the licensee in the presence of a royalty rate equal to Δ, but given no IDC burden. Note that the first derivative of Formula 3-29, with respect to the royalty rate Δ, is positive – proving that the DOL, and therefore the systematic risk and the required rate of return to the licensee, increases as the royalty rate increases.[11]

As before, the $DOL^{Royalty}$ is the degree of operating leverage experience by the licensee. However, the adjustment to beta is the $DOL^{Royalty}$ divided by $DOL^{No\ Royalty}$. This expression is as follows.

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[11] The first derivative is $\frac{SC}{(S(1-\Delta)-c)^2}$, which is positive.

(Formula 3-30) $$Royalty\ Beta\ Adjustment = \frac{DOL^{Royalty}}{DOL^{No\ Royalty}} = \frac{S(1-v-\Delta)[S(1-v)-C]}{S(1-v)[S(1-v-\Delta)-C]}.$$

Formula 3-30 is only slightly more complex than our beta adjustment for IDC relief. However, as we will see, the net adjustment for both fixed cost relief and taking on a royalty obligation involves some cancellation, and is not much more complex than Formula 3-29 – making application of the adjustment relatively straightforward.

c) Bringing it Together: Computing the Licensee's Cost of Capital

Now that we have the beta adjustments related to IDC relief and the royalty obligation, two steps are necessary in order to finally estimate the licensee's cost of capital. First, we must find the expression for the combined effect of IDC relief and the royalty. Second, we simply apply this adjustment to beta, and compute the unlevered CAPM-based cost of capital.

The expression for the combined effect of the two DOL adjustments is as follows.

(Formula 3-31) $$Licensee\ Beta = B^{LE} = B \times \frac{DOL^{Relief}}{DOL^{No\ Relief}} \times \frac{DOL^{Royalty}}{DOL^{No\ Royalty}}.$$

Where B^{LE} is equal to our "combined firm" unlevered beta, B, times the two adjustments. This reduces to a formula involving only licensee financial variables and an IDC cost estimate.

(Formula 3-32) $$B^{LE} = B\left(\frac{S(1-v)-C-I}{S(1-v)-C}\right)\left(\frac{S(1-v-\Delta)(S(1-v)-c)}{S(1-v)(S(1-v-\Delta)-C)}\right).$$

After cancellation we have

(Formula 3-33) $$B^{LE} = B\left(\frac{(S(1-v)-C-I)(S(1-v-\Delta)}{S(1-v)(S(1-v-\Delta)-C)}\right).$$

Formula 3-33 is our final formula for the beta coefficient for a licensee. This makes application of the CAPM quite easy. The licensee's cost of capital is simply.

(Formula 3-34) $$WACC^{LE} = r^f + B\left(\frac{(S(1-v)-C-I)(S(1-v-\Delta)}{S(1-v)(S(1-v-\Delta)-C)}\right)ERP,$$

where *ERP* is the equity risk premium.

D. The Licensor Side Mathematics

Now that we have the licensee cost of capital, the process for finding the licensor's cost of capital is very similar to the decomposition process for finding the required rate of return to residual profit. That is, we start with enterprise value formula for our notional combined firm, and decompose this into two separate licensor and licensee enterprise values. Then, given that the enterprise values for the licensor and licensee both involve profit functions, estimated growth rates, and discount rates, and given that we know the profit functions and estimated growth rates for both the licensor and licensee, and we know the licensee discount rate, we can solve for the licensor cost of capital that must hold in order for the licensee and licensor enterprise values to "add up" to the *EV* for the notional combined firm.

Beginning with the basic enterprise value relationship, we have

(Formula 3-35)
$$\frac{\pi}{r-g} = \frac{\pi^{LE}}{r^{LE}-g} + \frac{\pi^{LO}}{r^{LO}-g}\ ,$$

where, again, g is the growth rate, π is combined operating profit, π^{LE} and π^{LO} are the licensee and licensor operating profit, respectively, and the costs of capital for the combined firm, licensor, and licensee, are r, r^{LO}, and r^{LE}, respectively. Inserting the profit and cost of capital formulas that we've worked through thus far, we have

(Formula 3-36)
$$\frac{S(1-v)-C-I}{r-g} = \frac{S(1-v-\Delta)-C}{rf+B\left(\frac{(S(1-v)-C-I)(S(1-v-\Delta))}{S(1-v)(S(1-v-\Delta)-C)}\right)ERP} + \frac{S\Delta}{r^{LO}-g}.$$

Notice that we have all of the variables in formula 3-36 except for r^{LO}. This means that we can simply rearrange Formula 3-36 in order to get r^{LO} alone on the left-hand side of the expression, with all of the other elements of Formula 3-36 on the right-hand side. With some rearranging, we have the formula we are looking for:

(Formula 3-37)
$$r^{LO} = \left[\frac{S\Delta-I}{\left(\frac{S(1-v)-C-I}{r-g}\right)-\left(\frac{S(1-v-\Delta)-C}{\left[rf+B\left(\frac{(S(1-v)-C-I)(S(1-v-\Delta))}{S(1-v)(S(1-v-\Delta)-C)}\right)ERP\right]-g}\right)}\right] + g.$$

The astute reader may notice that this formula is identical in its basic structure to Formula 3-23 (the formula for the residual profit discount rate). That is, the formula for the residual profit discount rate reduced to $\bar{r} = \frac{\pi^{RN}}{(TEV-RTEV)} + g$, where TEV is total enterprise value and $RTEV$ is the "routine enterprise value."

Correspondingly, Formula 3-37 reduces simply to

(Formula 3-38) $$r^{LO} = \frac{\pi^{LO}}{(TEV-TEV^{LE})} + g,$$

where TEV^{LE} is the licensee's enterprise value. This highlights an important general result. Namely, when decomposing the WACC into its components, *any* cost of capital component is equal to the profit flow that corresponds to it, divided by the total enterprise value less the remaining profit flow, plus g. We will use this result elsewhere in this book.

E. An Example

Exhibit 3-9 provides a straightforward example of the application of our licensor and licensee cost of capital modeling. Exhibit 3-9 is developed as follows.

The first block of Exhibit 3-9 is labeled "Primary Financial Variables." While these are all assumed variables in Exhibit 3-9, their estimation is described below.

- Line 1 is the licensee's net sales. This is normalized to $100 in Exhibit 3-9 for simplicity, but need not be.
- Line 2 is the licensee's variable cost as a percentage of sales. This should be estimated using the licensee's average standard variable cost as a percentage of sales, or if such data are unavailable, the licensee's average COGS as a percentage of sales.
- Line 3 is the licensee's fixed cost. This is just total cost less variable costs. Fixed cost is normalized, relative to $100, in Exhibit 3-7. Importantly, for purposes of determining the licensee's cost of capital, we do not need to know his or her IDCs. Therefore, fixed costs for the licensee may include some or all of the licensee's investments in intangible assets.
- Line 4 is the licensor's IDCs. This is the investment, in total, that the licensor would have to make in order to support the licensee's sales and residual profit. Importantly, licensor IDCs should not be estimated as the incremental IDCs necessary for the licensor to engage in the license with the licensee – because often the incremental IDCs pertaining to a given are

low or zero. Rather, line 4 represents the "fully loaded" IDCs necessary to support the licensee. Put differently, line 4 is the IDC burden from which the licensee is relieved.

- Line 5 is the royalty rate, as a percentage of sales.
- Line 6 is the notional combined firm beta coefficient. This is the standard CAPM beta formulation, calculated in the standard way. As noted earlier, the beta for our "notional combined firm" should come from firms operating in the same, or a similar, industry, that both develop and exploit the intangible assets that the licensor and licensee are using. Given the importance of v and C to the licensor and licensee costs of capital, it is also useful if the comparable companies used to develop the combined firm beta have a similar cost structure to that of the combined licensor and licensee.
- Lines 7 and 8 are the risk free rate and equity risk premium. These data are widely available. In our practice, we generally rely on Ibbotson Associates, as well as other data sources, for the information contained in lines 7 and 8.
- Line 9 is the long run expected growth rate of the combined firm, over the term of the license. Given that the licensor and licensee likely have growth forecasts that exhibit varying rates of growth across years, line 9 can be estimated by solving for the compound annual growth rate at which the notional combined firm has the same TEV as it does given its actual, time varying, growth forecasts.

Importantly, with the exception of the licensee's sales, and the royalty rate, our estimates of all of the "primary financial variables" in the top section of Exhibit 3-9 will reside within ranges that are deemed to be reasonable. Because of this, the accompanying Excel™ files therefore contain a simulation feature that allows the user to input ranges for each of the key variables in the top block of the licensor-licensee WACC decomposition model. The simulation then produces frequency distributions for the licensor and licensee costs of capital.

Lines 10 through 20, which comprise the next three blocks of Exhibit 3-9, are simply applying the formulas that we've developed thus far. Notice that, as we would hope, lines 17 and 20 always add up to the total enterprise value in line 12.

Lines 11, 15, and 19 provide the notional combined firm cost of capital, the licensee cost of capital, and the licensor cost of capital, respectively. Notice that in Exhibit 3-9, the licensee's cost of capital is higher than the notional combined firm cost of capital – consistent with Exhibit 3-1. In Chapter 5 we discuss whether or not this is a necessary result.

One important feature of our model that is not immediately evident from Exhibit 3-9 is that a certain minimum "markup" on the licensor's R&D is necessary in order for the licensor to have a positive enterprise value. In other words, the fact that the licensor's IDCs ($10) are equal to 10 percent of sales does not imply that a royalty rate of, say, 11 percent of sales will produce a positive licensor enterprise value. Indeed, despite the fact that a royalty rate of 11 percent produces a profit of $1 per period for the licensor, the licensor's TEV is -$20.

Why is this? The reason has to do with the *licensee's* discount rate and resulting enterprise value. At royalty rates that are low relative to the IDCs from which the licensee is relieved, the licensee moves from an economic position equal to that of the notional combined firm (from a TEV of $244 the example given in Exhibit 3-9) to one in which his or her profit flow is nearly unchanged (at an 11 percent royalty rate, the licensee's profit moves from $20 if he or she bore the R&D to $19 given no R&D but an 11 percent royalty rate), but the licensee's risk profile is significantly different (at an 11 percent royalty rate the licensee's cost of capital moves from the notional combined firm capital cost of 11.2% to 10.2%. The second effect – the risk reduction – swamps the effect of slightly lower profit, and produces a licensee enterprise value that is greater than the TEV for the notional combined firm. By definition, this means that the licensor's TEV is negative. Not surprisingly, it turns out that below the minimum royalty threshold (the royalty rate at which the licensee is indifferent between licensing and investing in the IDCs), our formula produces a negative discount rate for the licensor.

Thus, our model produces a certain minimum royalty rate, below which the licensor has a negative enterprise value. In fact, the accompanying Excel files include a calculation of this minimum royalty rate. We leave it as an exercise for the reader to develop the formula the minimum royalty rate.[12]

[12] Hint: find Δ, such that the licensee TEV equals the combined firm TEV.

Exhibit 3-9

Example of WACC Decomposition Into Licensor and Licensee Capital Costs
Assuming Technology License (Licensor Bears R&D)

Line	Item	Value	Calculation
	Primary Financial Variables		
1	Sales (S)	$100	Assumed - Normalized to $100
2	Variable Cost as Percentage of Sales (v)	40%	Assumed
3	Fixed Cost (C)	$30	Assumed
4	Licensor IDCs (I)	$10	Assumed
5	Royalty Rate (Δ)	16%	Assumed
6	Notional Combined Firm Beta (B)	1.2	Assumed
7	Risk Free Rate (r^F)	4%	Assumed
8	Equity Risk Premium (ERP)	6%	Assumed
9	Expected Steady State Growth Rate	3%	Assumed
	Calculation of TEV for Notional Combined Firm		
10	Operating Profit for Notional Combined Firm (π)	$20	=Line 1 x (1-Line2) - Line 3 - Line 4
11	Cost of Capital for Notional Combined Firm (r)	11.2%	=Line 7 + Line 6 x Line 8
12	TEV for Notional Combined Firm	$244	=Line 10 / (Line 11 - Line 9)
	Calculation of Licensee Variables		
13	Licensee Beta Adjustment	1.05	Application of Formula 3-24
14	Licensee Beta	1.26	=Line 6 x Line 13
15	Licensee Cost of Capital	11.5%	=Line 7 + Line 14 x Line 8
16	Licensee Operating Profit	$14	=Line 1 x (1 - Line 2 - Line 5) - Line 3
17	Licensee TEV	$164	=Line 16 / (Line 15 - Line 9)
	Calculation of Licensor Variables		
18	Licensor Operating Profit	$6	=Line 1 x Line 5 - Line 4
19	Licensor Cost of Capital	10.5%	=(Line 18 / (Line 12 - Line 17)) + Line 9
20	Licensor TEV	$80	=Line 18 / (Line 19 - Line 9)
	Miscellaneous Variables		
21	Licensee Pre-Royalty Operating Profit	$30	=Line 16 + Line 1 x Line 5
22	Royalty as a Percentage of Licensee Pre-Royalty Operating Profit	53.33%	=(Line 1 x Line 5) / Line 21
23	Royalty as a Percentage of R&D	160%	=(Line 1 x Line 5) / Line 4

Chapter 3 Technical Appendix

The following diagram is taken from Ruback (1992), and shows the derivation of enterprise value using capital cash flows versus free cash flow.